Study to Gauge the Performance of Queuing System Using Multi-Server Queuing Model (M/M/S) in a Private Hospital Outpatient Department

By

Payal Mittal

Dissertation submitted in partial fulfillment of the requirements of the degree

MBA Hospital and Health Management

Academic year, 2018-2020

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56329

Internship Completion Certificate

This certificate is awarded to

Dr. Payal Mittal

In recognition of having successfully completed her
Internship in the department of **Operations**
and has successfully completed her Project on

Study to gauge the performance of Queuing system using Multi-server queuing model (M/M/S) in a private hospital outpatient department

Internship Date: 10th February 2020 to 31st May 2020

We wish her all the best for future endeavors.

For Burjeel Hospital, Abu Dhabi

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CERTIFICATE BY SCHOLAR

This is to certify that the dissertation titled "To gauge the performance of Queuing system using Multi-server queuing model (M/M/S) in a private hospital outpatient department" and submitted by Dr. Payal Mittal Enrolment No U/01/2018-20/0796 under the supervision of Dr. P.R. Sodani, Pro-President and Dean, Institute of Health Management Research, IIHMR University, Jaipur for award of MBA in Hospital and Health/ Pharmaceutical / Rural Management in Health and Hospitals of the University carried out during the period from 10th February 2020 to 10th May 2020 embodies my original work and has not formed the basis for the award of any degree, diploma associate ship, fellowship, titles in this or any other institute or other similar institution of higher learning.

Name: Dr. Payal Mittal (BDS, MDS)

MBA Health and Hospital Management

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"The dream begins with a teacher who believes in you, who tugs and pushes and leads you to the next plateau, sometimes poking you with a sharp stick called 'truth'."

DAN RATHER

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Date: 20th June

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ABSTRACT

AIM AND OBJECTIVE: Significant variation is observed in out-patient department; patient arrival rates necessitate the adjustment of server number to optimize the timely care of patients. Henceforth, this study aims to further gauge the performance of an outpatient department in a private hospital. In particular, the study aims to investigate the current efficiency of queuing system of the said department and propose potential solution for performance improvement. The study implemented the queuing theory using multi server queuing model (M/M/S) to measure the performance of waiting line. The study also evaluated the effectiveness of a queuing model in identifying ideal number of server for reducing the fraction of patients due to long waiting time.

METHODOLOGY: Descriptive-analytical study was carried out in out-patients department located at Burjeel Hospital Abu Dhabi (UAE).Data was collected for a period of four weeks i.e. from 15th February to 15th March 2020. A total of 8677 samples were included in study using the purposive sampling method. The per hour arrival data that was observed was matched with the data extracted by the hospital system (SAP) and the extracted data were used to map the arrival pattern of patients in department. A Fishbone analysis was carried out to show cause and affect relationship that give rise to certain problems. The study focused on application of multi-server queuing model to measure the performance of waiting line. A computational work (QM software, Operations research) was then presented to value the operating characteristic of the model. The finding exhibit the performance of existing queuing system and the need to re-examine the number of sever required to efficiently handle the patient flow. In all, study was a stepping stone as a decision making aid for manager to particularly relook and improve the existing system in tandem with center commitment for better patient management and minimal patient complaints.

RESULT: The result showed that maximum waiting time was observed from entry till the registration thereby delaying the entire waiting time in the system. The registration process as considered first patient-provider encounter, therefore the entire focus of the study was on the number of ideal server required to efficiently handle the average arrival pattern in accordance with service rate of server. The finding shows that maximum waiting time was noted from 7-10.0 am, 12-13pm, 16-17pm, 19-20pm, and 20-21 pm and was observed as peak hours. According to the results, the number of server required for registration can be changed hourly and daily or can be adjusted from other clinics during peak hours. The average number of server required is four compared to previous queuing model where on average there were only two. According to the model outputs in every section, a briefing is projected looking the marginal changes for every part with respect to the average number and the average waiting time within the queue.

CONCLUSION: In a current competitive business scenario, there is always a growing demand to make correct management decisions at all times and one of those decision is on waiting time management which is also considered as one of the measure of service quality by patients. In an environment in which out-patient department are often understaffed, analyses of arrival patterns and the use of queuing models can be extremely useful in identifying the most effective allocation of staff for timely and efficiently management of patients.

KEYWORDS: *OPD*-Out- patients department, *TAT*- Turnaround time, *M/M/S*- Multi-server queuing model, *QM*- Queuing management software tool, *SAP*- System application and product, *FCFS*- First-come-first-served, *FIFO*- First-in-First-out

TABLE OF CONTENT

S.NO	ABBREVIATION
1	OPD-Out- patients department
2	TAT- Turnaround time
3	M/M/S- Multi-server queuing model
4	QM- Queuing management

CHAPTER 1

INTRODUCTION

Queues or waiting lines are experienced commonly in day to day life whereby array of business providers require their customers to wait in line, such as for telephone services, supermarkets, banking or one of the another fastest growing sector i.e. Healthcare. The methods of studying these line formations and different ways to manage them is termed as Queuing theory. Basically, it is a mathematical way of analysing the system, which renders service to random demands and deals with one of the common &irksome experience, waiting. The example of telephone exchange was mentioned because the first problem of queuing theory was raised by innumerable number of calls and it was Erlang who first fixed this congestion issue at the commence of 20th century. Queuing problem is basically defined when the service rate of facility provider lags behind the service rate to the customers (1). The businesses who are highly challenged with these waiting lines are Healthcare. It is one of the fastest growing industry which demands perfection, top notch services and the optimal capacity utilization. Waiting lines in healthcare setting are most commonly encountered in the out-patient department of hospital. Waiting time is considered as most critical factors in outpatient clinical settings, as it has direct impact on the recipient's health and their satisfaction level. Outpatient department staffs should be involved in various activities including information management, appropriate resources reallocation, accurate use of data and the timely execution of processes to maintain proper patient flow in the department (2, 3). Producing such faster services with very minimal waiting time for the outpatient department is always been the goal of many service providers in recent years (4, 5).

For businesses and organizations to grow there's always a requirement to form management decisions in the least possible time. Thus, attention on waiting time management is crucial for a competitive business market whereby minimizing the waiting time is considered as a measure of service quality taken into account with important factor by customers. The performance measurement of health care systems is vital for improving the service provided to the patients. Thus, modeling the flow of patient's and repairing environment measures under the conditions of uncertain demand can produce a dramatic improvements in medical performance, patient satisfaction, and price efficiency of the healthcare system (6).

There are three methodologies that are mostly used to analyse health care process: A. Simulation modelling, B. Flowcharting tools, and C. Queuing models. A simulation model is used for detailed representation of complex, dynamic behaviour; however, building a model is extremely time consuming and requires the experienced modellers. The flowcharting tools basically represent the process structure but the downside are they neither able to represent the variation of the process nor they support the quantitative analysis of process design alternatives. The last queuing models are used to understand the dynamic behaviour and can be built in enough time period compared to simulation model which is time consuming (7). Considering the importance of the above facts, the first phase of study focuses on the current queuing pattern and waiting lines in department and root and cause analysis for long waiting time. The second phase of study concentrate on application of queuing model to find the optimal server required to reduce the fraction of patient due to long waiting time and improve the performance in out-patient department.

SIGNIFICANCE OF STUDY

Outpatient departments (OPDs) are considered as the shopping window to hospital services and play an important role to build patient's impression towards hospital. It is the first service encounter of patient and the provider which often influences the patient's perception towards the hospital and therefore a checkpoint of utmost importance to ensure that OPD services provide an exceptional experience for their clients. Though, the hospitals have much succeeded in providing advancement in medical and technical area for rendering more effective treatments, but they are still struggling with delays and inefficiencies, thereby presenting a good platform for research in important scientific fields like queuing management. In a current competitive business scenario, there is always a growing demand to take correct management decisions every times and one of those decision is on waiting time management which is considered as one of the measure of service quality by patients. It is often seen that patients make choices of either leaving the hospital without being treated or switch to other hospitals due to long waiting queue thereby affecting the brand image of organization. Considering all the above factors in priority and taking into account the long waiting time in department, the project was undertaken to formulate a queuing model for optimal utilization of the current system and improving the performance of the department.

CHARACTERISTIC OF QUEUING MODEL

The waiting line model was originated from the work of Erlang in 1917 the time when he published a paper on telephone traffic theory (8). His work has inspired many subsequent studies on the queuing theory and relevant application problems in many different kinds of industry such as healthcare and engineering-related fields (9). There are two basic approaches to the queuing problems; (1) analytical methods using either single server model (M/M/1) or the multi-server model (M/M/s) and the simulation method (10, 11). Queuing Theory (QT) is the mathematical way of studying the queues or waiting lines. QT can be extrapolated to numerous faculties, yet most of prior studies are documented mainly in the field of Operations Research, Probability, & Management Science. Queuing theory has been applied to simulation models to help in deciding the numbers of hospital servers, proper resource utilization and to curtail waiting time. QT has been much extrapolated in industrial settings to analyze how resource- constrained systems responds to diversified demands. And thus, it may be a natural fit model to deal with patient flow in a medical setting (12, 13).This theory allows the mathematical breakdown of several interconnected process that include arriving in the queue, waiting time and being served at the front of the line. Queuing theory enables the computation of several performance measures. (14). A queuing system is compared to a structure where “customers” arrive to a bank of “servers” and require some sort of services from one of them. “Customer” is referring to the entity who is expecting the service; similarly, a “server” is a man or machine that provides the service (15). In a hospital setting, queuing system incorporates patient’s arrival time, waiting time, utilization of service, and exiting the system after receiving the service (16).

TYPES OF QUEUING SYSTEMS

Basically, queuing system is categorized into four types. According to Lapin (1981)**,** Queuing system incorporates the following. (17)

a. **Single-server, Single-phase system:**
It is defined when single queue of customers are to be served by a single service facility (server) one after the other.

b. **Single-server, Multiple-phases System:**
It is defined when there's still a single queue but customers receive more than one kind of service before departing the queuing system.

c. **Multiple-servers, Single-phase System:**
Queuing system is characterized by a situation whereby there is a more than one service facility (servers) providing identical service but drawn on a single waiting line. Example is registration for physician consultation in department

d. **Multiple servers, Multiple-phases System:**
According to Singh (2007)**,** this type of service is usually seen during a hospital setting, multi-specialty outpatient clinics, patient first form the queue for registration, and then he/she is triage for assessment, then for consultation and finally exits from the system or triage to different provider. (18)

Performance Evaluations of Queuing Systems

According to Hillier and Lieberman, (2005) performance parameters in a queuing system are categorized as follow (19):

a. System Utilization (p): System Utilization is an important vital measure. It measures the standard time for which system is busy. It's the ratio of system capacity utilized to the available capacity. A zero value in the system means there's nobody within the system. On the contrary, a system utilization of ≥ 1 depicts that there are infinite number of people in the queue. This suggests that the available number of servers cannot cope up the approaching demand. Thus, additional measure needs to be taken (20). System Utilization should be greater than 0 or less than 0.8 (Egolum, 2001).

b. Mean Number within the system (Ls): It is defined as the standard number of system users (entities) that includes number of patients in the queue or patient waiting for the service.

c. Mean Number in Queue (Lq): It is characterized by the standard or anticipated number of patients within the waiting line, or the patients looking for getting their problem resolved.

d. (Wa): The average time of waiting for an arrived patient but not immediately served.

e. Mean time in System (Ws): It is defined by the initial or average waiting time that the patients will stay within the system. It defines the quality time that the service will start thus called as standard service time.

f. Mean time in Queue (Wq): It is characterized by initial avg. time that the patients will spend in the queue and avg. time patient will wait for the service to start.

h. Probability of waiting (Pw): This is the probability that the patient will wait for its service to start out.

CHAPTER 2

REVIEW OF LITERATURE

Linda V. Green, James F. Giglio, Joao Soares et al (2006) conducted "study to measure the effectiveness of a queuing model in identifying and curtailing fraction of patients who leave without being seen". The study stated that analyzing the arrival patterns and utilization of queuing models can be significantly useful in reconditioning effective reallocation of staff members. A significant change recognized by the model was that the generous staff increment was required to deal with the morning surge. The observations from these analysis serves as guiding principles in developing new schedules for providers. (21) The study result was found in accordance with our result suggesting optimal increase in the number of servers during peak hours can lead to desirable performance improvement. Other study conducted by Sam Afrane, Alex Appah (2014) was to understand the queuing theory and its application to manage the queuing problem at OPD of AngloGold Ashanti hospital in Obuasi, Ghana. The study highlighted that the system performance can be improved by optimizing the system capacity by increasing the number of doctor from eight to five in number. In addition the study also concluded that application of queuing theory and model to the existing queuing and capacity challenges can aid in efficient decision making which will contribute the optimal performance improvement. (22). The study result were in accordance with our result suggesting the increase of sever from average two to minimum of average of four during peak hours can improve the efficiency of the department. Another study conducted by Dr. S. K. Tiwari1, Dr. V. K. Gupta et al (2015) on multi server model to find the minimum expected cost. The study has highlighted the various common queuing conditions and showed a variety of mathematical models to analyse the waiting lines backing certain assumptions.This paper has introduced the concepts of queuing models for optimal utilization of resources and for efficiency improvement of the system. It shows that how linear programming, and mathematical analysis, can be best used to estimate the actual performance measures of system. (23). Our study in concurrence with the above study applied queuing theory using QM software, an operational research tool for understanding the waiting line and to implement solution for optimal utilization and efficiency improvement in out-patient department. Another study by Hendrik Lamsali, Mohamed Najib Salleh (2016) conducted on to investigate the current efficiency level of a queuing system of the out-patient department and to propose potential solutions for same. To measure the performance of waiting lines, multi-server queuing model was applied, and a computational work was presented to value the operating characteristic of the model. The finding of the study suggested that improving the performance can be achieved through rescheduling of doctor duty hour for the optimal utilization and for reducing the waiting time. It further concluded the benefits of application of queuing model for better decision making that aid the administrator to re-look and improve the existing queuing system for better customer satisfaction and reduce patient turnover (24). The other study by P. A. Dilrukshi, G. H. Lanel, H. D. Nirmanamali et al (2016) conducted a study "to identify the benefits of application of queuing model for reducing the patients' waiting time at the OPD of the National Hospital of Sri Lanka. The study result were in congruence with our result concluding the importance of queuing model for managerial decision making by highlighting the facts that modification are required on number of sever (doctor)for consultation and pharmacy to be changed on hourly or daily interval while avg. number of doctor should be assigned to work hourly basis for all days. The pharmacy result showed that 4 or 5 counters are ideally required in order to combat the waiting time. (25). Another study by Tochukwu A. Ikwunne, Moses O.

Onyesolu (2016)conducted a study “To identify ideal numbers of server at minimal cost which include waiting period and the service costs for reducing the patients’ bottleneck in the hospital to minimum using the QT model”. The results highlighted that the avg. length of queue, waiting period and excess utilization of specialist doctors can be reduced at an favorable number of server level with minimal cost compared to previous with high cost. Therefore, study proposes on refocusing on the ideal no. of server for improving the overall care and meets the expected patients’ needs. (26). Other study by Obulor R., Eke B. O (2016)conducted a study to reduce the waiting time in OPD and patient turn over in one of the specialists, teaching and general hospital of Nigeria. The study suggested that by implementing the effective queuing model for proper scheduling can reduce the waiting time, ideal server time (doctors) and can also impact the overall efficiency of the department as well as on patient satisfaction (27). Another study by Disha Chandra (2017) conducted a systematic survey on different type of issues that contribute to increase in waiting time and the techniques used to deal with such structural and functional problem for reducing the waiting time for out-door patients. The study concluded that queuing theory application is an edge in reducing the waiting time and for improving the overall patient satisfaction of the patients (28). The other study by A. NellaiMurugan, S.Vijayakumari Saratha (2017) analyzed the queuing characteristics at the specialist clinicusing M/M/S queuing model. The study also included waiting period and service costs with a focus to identify the ideal number of server required for performance improvement. The study concluded that the avg. length of queue, waiting period of patients can be effectively managed by increase the service capacity of doctors from 5 to 9 with incurring the high service and waiting cost (29).A.H. Nor Aziati, Nur Salsabilah Binti Hamdan (2018)in their study identified the waiting period involved during the entire process flow and applied queuing theory to improve operational performance and service quality. The result showed that application of multiple server channel patient flow simulation model concluded that the waiting time is in accordance with Ministry of Health patient charter. The study also showed that application of queuing model can be an effective tool in decision making especially for on the staff needs for favorable performance of the department (30). Other studies conducted by Onoja Anthony, MsughKembe (2018) by analysing the queuing distinctive character at ante-natal care clinic of teaching Hospital through application of M/M/S Model and have also focussed on waiting and the service cost in order to gauge the optimal service level at minimum cost. The result of the analysis explicitly showed that the avg. length of queue, average waiting period of pregnant women in clinic and the overworking of server (doctors) could be reduced to ideal server number of 6 with minimal cost in contrast to previous server level of 4 with high total cost. This model will aid in efficient decision and for other policies founder to unravel the M/M/S queuing problem in an area where capacity and limited resource gives rises to longer waiting and services time and affect patient satisfaction.(31)

CHAPTER 3

RESEARCH METHODOLOGY

RESEARCH QUESTIONS

1. What is the process flow of the outpatient department?
2. What is the root and cause of long waiting time in system?
3. What is the current pattern of patient arrival?
4. What is the current service rate?
5. How much is the system utilization rate?
6. What is the average duration patients wait in the system and queue
7. What is the average no. of patients in the system and queue?
8. What is the number of servers required to reduce waiting time?

OBJECTIVES

1. To study the process flow of the out patients department.
2. To study the existing queuing model in department
3. To undergo root and cause analysis for long waiting time.
4. To study the arrival and service rate in out-patient department.
5. To study the system utilization rate
6. To identify the av. time patient, wait in the queue and system
7. To identify the average no. of patients in the queue and system
8. To identify the ideal number of servers required to reduce the waiting period
9. To propose an alternative queuing model for performance improvement

METHODOLOGY

Descriptive-analytical study was carried out in out-patients department located at Burjeel Hospital Abu Dhabi (UAE).Data collection period was of four weeks i.e. from 15th February to 15th March 2020. Data was collected on hourly basis for studying the service rate of the server and to observe the arrival pattern. A total of 8677 samples were included in study using the purposive sampling method. The method employed for data collection for arrival pattern and service time was direct observation, personal interview with staffs. The checklist was prepared for recording the patient's arrival time, service time of each patient by server and waiting time at different nodes involved till consultation. The per hour arrival data that was observed was matched with the data extracted by the hospital system (SAP) and the extracted data were used to map the arrival pattern of patients in department. A Fishbone analysis was carried out to show cause and effect relationship that give rise to certain problems (32). To evaluate the performance of out-patient department, a total of 8677 sample were included in the study and queuing model was applied, a concept of queuing work with the mathematical computational modelling and analysis of existing systems lay out service to arbitrary demands. The following parameter of queuing model such as capacity utilization rate, avg. time the patients takes in the queue and the system, avg. no. of patients waiting in the system &the queue and the number of ideal server was calculated using QM software for Windows (Operational Research). Generally the queuing model constitutes the concrete lay out by identifying the ideal number and sorting of the servers that provide service to the patients. The following were the inclusion and exclusion criteria identified for project:

INCLUSION CRITERIA

a. The department with highest flow of patients were included in the study namely Family Medicine, Internal Medicine, Endocrinology and Pulmonology
b. Patients who waited in queue on arrival
c. Patients followed first come first basis protocol based on the token number taken through token vending machine

EXCLUSION CRITERIA

a. Patients who abandon the queue were excluded
b. Any patients who did not adhere the protocol of first come and first serve protocol
c. Patients who didn’t take token from vending machine were excluded
d. Data of patient flow for Weekend (Friday) were excluded

LIMITATION OF STUDY

Service and waiting cost could not be included in study due to the financial data privacy.

SAMPLING DURATION

15th February 2020 to 15th March 2020

CHAPTER 4
RESULT AND DISCUSSION

THE QUEUING MODEL

The model applied in research methodology is: M/M/S model; Amulti-server, single-phase queuing system represented by a condition where there is more than 1 server and providing akin services but still wan on single waiting line. This system is often practiced at the out-patient departments where usually a single waiting line and several servers are present. Due to large population, the length of queue is never ending process and the entire queuing system is multi-channel therefore known as multi server queuing model.It is presumed that influx of patients in department follow Poisson's Probability Distribution at a mean variety of λ patients/hour, whereas the service times are exponentially distributed; with an Av. of μ/hour and no. of servers (33). It is further considered that the Queuing system follows First Come, First Served (FCFS pattern) principle by the respective Servers {Guest service executives}.

The following parameters were evaluated for this model:

- ⇒ Mean customers arrival count: λ
- ⇒ Mean service rate: μ
- ⇒ Utilisation factor{ρ}= λ / μ
- ⇒ Probability of receiving zero customers, $0 = 1 - p$
- ⇒ Probability of having n number of customers, $n = P0\ \rho n = \{1 - p\}\rho n$
- ⇒ Av. no. of customers in the server: $Ls = \lambda / \lambda - \mu$
- ⇒ Av. no. of customers in the queue: $Lq = Ls - \lambda / \mu$
- ⇒ Av. time patients spent in the server, $Ws = 1 / \mu - \lambda$
- ⇒ Av. duration of patients spent in the queue, $Wq = p / \mu - \lambda$

PATIENT FLOW:

Figure1.1. Patient flow in the out-patient department

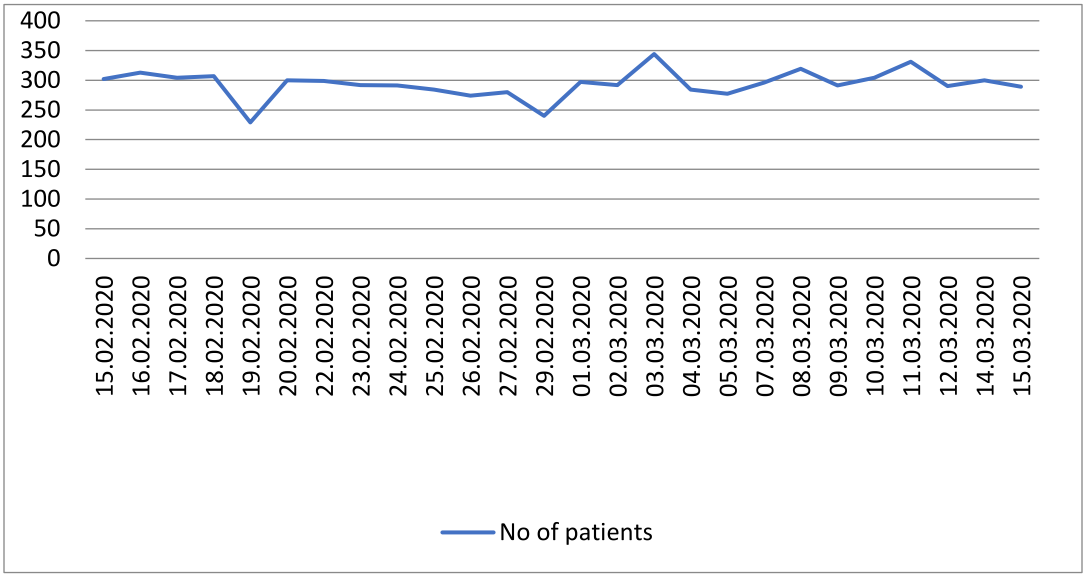

Figure.1.2. The figure shows the number of patients flow at out-patients department

The figure showed the increasing number of patient flow throughout the study. The one of the reason of increasing flow was identified as due to the pandemic situation that is COVID-19.The sudden outbreak had led change in the equilibrium of demand and supply. {Fig.1.2}

Timing	**Total no of patients**	**No of servers**	**Av.{Arrival rate per hour}**	**Av.{service rate per hour}**	**System Utilization rate**	**Waiting time in queue {minutes}**
7-8am	217	1	8	10	0.8	37
8-9am	466	2	33	17	0.97	114
9-10am	593	2	42	22	0.95	51
10-11am	659	3	48	19	0.84	34.8
11-12am	727	4	52	38	0.34	10
12-13pm	723	4	51	18	0.94	29
13-14pm	679	4	48	41	0.29	15
14-15pm	712	3	50	25	0.67	10
15-16pm	636	3	45	21	0.71	20
16-17pm	648	2	48	26	0.92	26.5
17-18pm	595	3	43	24	0.6	13
18-19pm	553	3	39	22	0.59	10
19-20pm	455	2	32	17	0.94	54
20-21pm	441	2	32	18	0.89	27
21-22pm	342	1	24	25	0.96	24
22-23pm	160	1	11	12	0.92	24
23-00pm	69	1	5	11	0.45	22
00-1.00pm	2	1	1	9	0.11	19

Table.1.1 The current queuing characteristic for registration at out-patients

Table 1.1 illustrates the current queuing condition based on the Av. no. of patients per hour and Av. no. of patients served by server per hour. Data focuses on Av. waiting time required to undergo first process of consultation that is registration. Registration process at out- patients department is considered as first service encounter of patients with healthcare provider and thereby had great impact on patient experience and satisfaction. The above data were analyzed using Queuing software {QM} tool of operational research and the result obtained were rounded to nearest absolute values. The result shows that maximum waiting time was observed from 7-100 am, 12-13pm, 16-17pm, 19-20pm, and 20-21pm and was observed as peak hours. It was observed that even after optimal system utilization there were long waiting queues, therefore, along with QM software application, one of the quality tools that are Fish Bone analysis was also carried out to understand the cause and effect relationship that give rise to certain problems. {Fig 1.3}

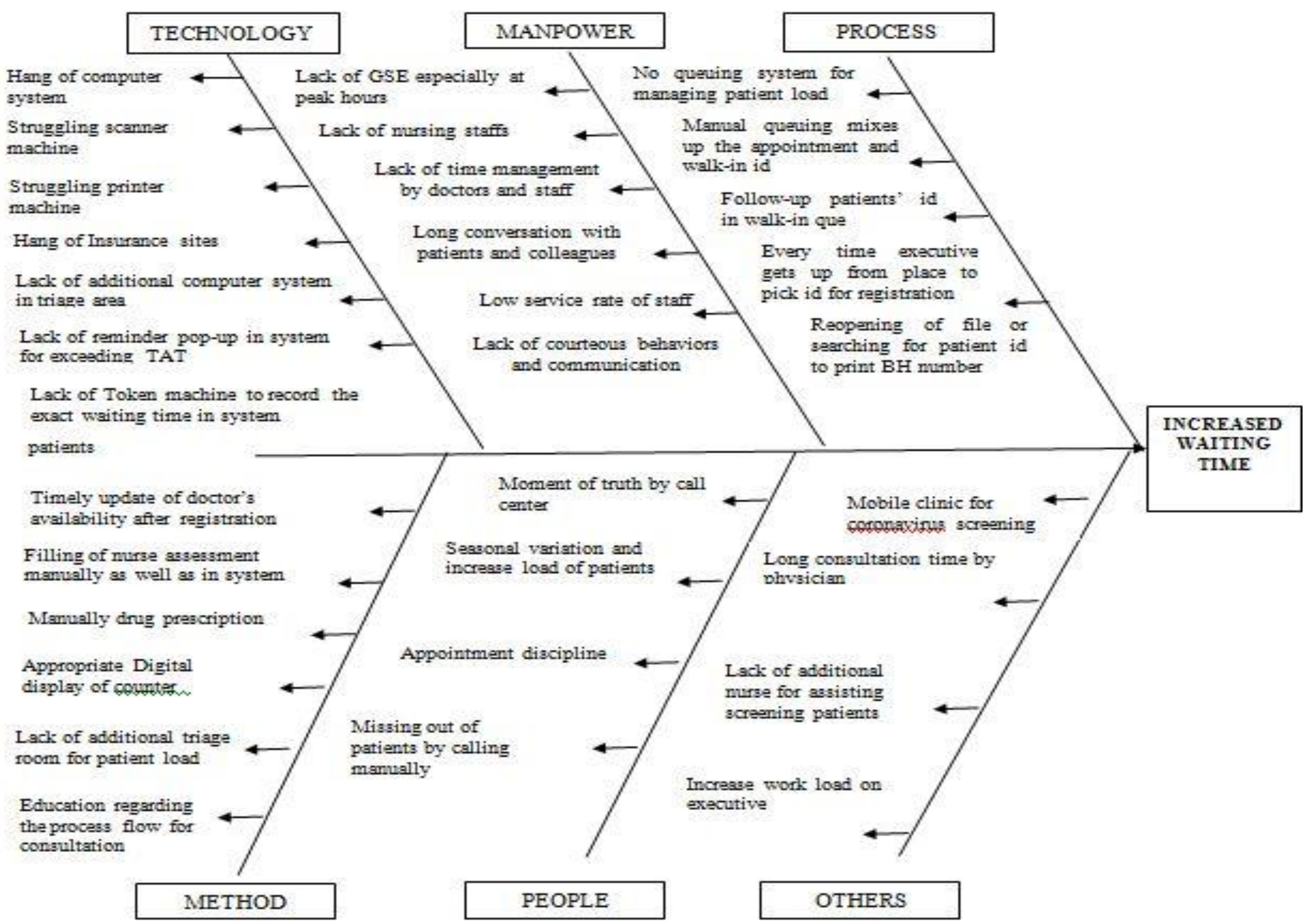

Figure1.3. Fish Bone analysis diagram

Considering the above factors, we focused on one of the reasons that are to identify the number of ideal server required for registration for performance improvement in out-patient department. It was observed that the maximum time was invested in registration process and due to delay in the first step of consultation; the Av. total time from entry of patients in department till the consultation was recorded high. Therefore, the queuing theory was applied on hourly data for calculating other parameter of queuing model such as no. of patients in the system and queue, Av. waiting period in the queue and Av. waiting period in the system and number of servers required to improve the overall performance.{Table 1.2}

Table.1.2 Time-wise application of QT (queuing method) to understand the queuing model

		Results		**Minutes**	**Total {minutes}**
		Av. server utilization	0.8		
Time	**7-8am**	Av. No. of customers in queue {L_q}	3.2		
Arrival rate {λ}	8	Av. No. of customers in the system{L}	4		
Service rate { μ}	10	Av. waiting duration in the queue{W_q}	0.4	13	**37**
Number of servers{s}	1	Av. duration in the system{W}	0.5	30	**60**
		Probability of system is empty {P_0}	0.2		
		Results		**Minutes**	**Total {minutes}**
Timing	**8-9am**	Av. server utilization	0.97		
Arrival rate { λ}	33	Av. No of customers in the queue{L_q}	31.55		
Service rate {μ}	17	Av. No of customers in the system{L}	33.49		
Number of servers{s}	2	Av. waiting period for the queue{W_q}	0.96	57.37	**114**
		Av. time period in the system{W}	1.01	60.9	**120**
		Probability of system is empty {P_0}	0.01		
		Results		**Minutes**	**Total {minutes}**
Timing	**9-10am**	Av. server utilization	0.95		
Arrival rate { λ}	42	Av. no. of customers in the queue{L_q}	19.58		

Service rate {μ}	22	Av. no. of customers in the system{L}	21.4 9		
Number of servers{s}	2	Av. waiting period in the queue{W_q}	0.47	27	**51**
		Av. duration in the system{W}	0.51	30	**60**
		Probability of system is empty {P_0}	0.02		
		Results		**Minutes**	**Total {Minutes}**
Timing	**10-11am**	Av. server utilization	0.84		
Arrival rate {λ}	48	Av. No. of customers in the queue{L_q}	3.82		
Service rate {μ}	19	Av. No. of customers in the system{L}	6.35		
Number of servers{s}	3	Av. waiting duration in the queue{W_q}	0.08	30	**34.8**
		Av. time period in the system{W}	0.13	40	**47.8**
		Probability {% of time} system is empty {P_0}	0.04		
		Results		**Minutes**	**Total {Minutes}**
Timing	**11-12pm**	Av. server utilization	0.34		
Arrival rate {λ}	52	Av. No. of customers in the queue{L_q}	0.03		
Service rate {μ}	38	Av. No. of customers in the system{L}	1.40		
Number of servers{s}	4	Av. waiting duration in the queue{W_q}	0.05	5	8
		Av. time period in the system{W}	0.08	8	12.8
		Probability {% of time} system is empty {P_0}	0.25		
		Results		**Minutes**	**Total {minutes}**
Timing	**12-13pm**	Av. server utilization	0.94		
Arrival rate {λ}	51	Av. No. of customers in the queue{L_q}	15.2 5		
Service rate {μ}	18	Av. No. of customers in the system{L}	18.0 8		
Number of servers{s}	3	Av. waiting period in the queue{W_q}	0.30	17	**29**
		Av. time period in the system{W}	0.35	21	**39**

		Probability {% of time} system is empty {P_0}	0.01		
		Results		**Minutes**	**Total {Minutes}**
Timing	**13-14pm**	Av. server utilization	0.29		
Arrival rate {λ}	48	Av. No. of customers in the queue{L_q}	0.01		
Service rate {μ}	41	Av. No. of customers in the system{L}	1.18		
Number of servers{s}	4	Av. waiting duration in the queue{W_q}	0.00	0.02	0.02
		Av. time period in the system{W}	0.02	1.48	3
		Probability {% of time} system is empty {P_0}	0.31		
		Results		**Minutes**	**Total {Minutes}**
Timing	**14-15pm**	Av. server utilization	0.67		
Arrival rate {λ}	50	Av. No. of customers in the queue{L_q}	0.89		
Service rate {μ}	25	Av. No. of customers in the system{L}	2.89		
Number of servers{s}	3	Av. waiting duration in the queue{W_q}	0.02	1.07	2.27
		Av. time period in the system{W}	0.06	3.47	6.5
		Probability {% of time} system is empty {P_0}	0.11		

		Results		Minutes	Total {Minutes}
Timing	**15-16pm**	Av. server utilization	0.71		
Arrival rate {λ}	45	Av. No. of customers in the queue{L_q}	1.28		
Service rate {μ}	21	Av. No. of customers in the server{L}	3.43		
Number of servers{s}	3	Av. waiting duration in the queue{W_q}	0.03	1.7	3
		Av. time period in the system{W}	0.08	4.57	9
		Probability {% of time} system is empty {P_0}	0.09		
		Results		Minutes	Total {Minutes}

Timing	**16-17pm**	Av. server utilization	0.92		
Arrival rate {λ}	48	Av. No. of customers in the queue{L_q}	10.6 3		
Service rate {μ}	26	Av. No. of customers in the system{L}	12.4 8		
Number of servers{s}	2	Av. waiting duration in the queue{W_q}	0.22	13.29	**26.5**
		Av. time period in the system{W}	0.26	15.6	**31**
		Probability {% of time} system is empty {P_0}	0.04		
		Results		Minutes	Total {Minutes}
Timing	**17-18pm**	Av. server utilization	0.60		
Arrival rate {λ}	43	Av. No. of customers in the queue{L_q}	0.52		
Service rate {μ}	24	Av. No. of customers in the system{L}	2.31		
Number of servers{s}	3	Av. waiting duration in the queue{W_q}	0.01	0.73	1.5
		Av. time period in the system{W}	0.05	3.23	6.5
		Probability {% of time} system is empty {P_0}	0.15		
		Results		Minutes	Total {Minutes}
Timing	**18-19 pm**	Av. server utilization	0.59		
Arrival rate {λ}	39	Av. No. of customers in the queue{L_q}	0.50		
Service rate {μ}	22	Av. no. of customers in the system{L}	2.27		
Number of servers{s}	3	Av. waiting duration in the queue{W_q}	0.01	0.76	1.5
		Av. time period in the system{W}	0.06	3.49	7
		Probability {% of time} system is empty {P_0}	0.15		
		Results		Minutes	Total {Minutes}
Timing	**19-20pm**	Av. server utilization	0.94		
Arrival rate {λ}	32	Av. No. of customers in the queue{L_q}	14.6 0		
Service rate {μ}	17	Av. No. of customers in the system{L}	16.4 8		

Number of servers{s}	2	Av. waiting duration in the queue{W_q}	0.46	27.38	**54**
		Av. time period in the system{W}	0.52	30.91	**60**
		Probability {% of time} system is empty {P_0}	0.03		
		Results		Minutes	Total {Minutes}
Timing	**20-21 pm**	Av. server utilization	0.89		
Arrival rate {λ}	32	Av. No. of customers in the queue{L_q}	6.69		
Service rate {μ}	18	Av. No. of customers in the system{L}	8.47		
Number of servers{s}	2	Av. waiting duration in the queue{W_q}	0.45	0.1	**27.1**
		Av. time period in the system{W}	0.30	20	**38**
		Probability {% of time} system is empty {P_0}	0.06		
		Results		Minutes	Total {Minutes}
Timing	**21-22 pm**	Av. server utilization	0.96		
Arrival rate {λ}	24	Av. No. of customers in the queue{L_q}	23.04		
Service rate {μ}	25	Av. No. of customers in the system{L}	24		
Number of servers{s}	1	Av. Waiting duration in the queue{W_q}	0.21	12.5	24
		Av. time in the server{W}	0.26	15.8	28
		Probability {% of time} system is empty {P_0}	0.04		
		Results		Minutes	Total {Minutes}
Timing	**22-23 pm**	Av. server utilization	0.92		
Arrival rate {λ}	11	Av. No. of customers in the queue{L_q}	10.08		
Service rate {μ}	12	Av. no. of customers in the system{L}	11.00		
Number of servers{s}	1	Av. waiting duration in the queue{W_q}	0.4	0.02	24.02
		Av. time period in the server{W}	1	0.00	60

		Probability {% of time} system is empty {P_0}	0.08		
		Results		Minutes	Total {Minutes}
Timing	**23-00 pm**	Av. server utilization	0.45		
Arrival rate {λ}	5	Av. No. of customers in the queue{L_q}	0.38		
Service rate {μ}	11	Av. No. of customers in the system{L}	0.83		
Number of servers{s}	1	Av. Waiting duration in the queue{W_q}	0.08	4.55	11
		Av. time period in the server{W}	0.17	10	22
		Probability {% of time} system is empty {P_0}	0.55		

The above data shows during the peak hours as lightest modification in the allocation of server using queuing software can promise for better performance. The Av. TAT for registration was standardized as 5 minutes. The observation highlighted that the following were the peak hour where in spite of maximum capacity utilization, highest waiting time was observed: 7-8 am, the capacity utilization was found to be 80%, Average no. of patients in queue was 3.2, the available no. of server was one and waiting time in queue was noted as 37 minutes; 8-9 am the capacity utilization was found to be 97%, Av. number of patients who are taking service or still in queue was 31.55. The Av. waiting time in queue was found to be 114 minutes and available number of server was two in number; 9-10am, system utilization was found to be 95%, Average no. of patients in queue was 19.58 and waiting time was 51 minutes and available number of server was two; 16-17 pm shows 92% capacity utilization, Av. number of patients in queue was found to be 10.63, available number of server was two and waiting time was noted as 26.5 minutes. Another round of peak hours was observed in evening hours from 19-20pm, capacity utilization was found to be 94%, Av. number of patients in queue was 14.60, available no. of sever was two and waiting time calculated was 54 minutes. The timing from 20-21 pm, capacity utilization was found to be 89%, Av. number of patients in queue was 6.69, available no. of sever was two and waiting time calculated as 27 minutes.

Using Multi-server Queuing model; the waiting duration in the queue, the length of the queue and all the other variables were computed by altering the no. of channels for each single hour. When the no. of servers was increased, the span of the queue can be shortened and waiting period decreases. As per prevailing no. of channels for the registration, the queuing system is inadequate. The alternative queuing model was proposed by modifying the no. of servers during peak hours and result were analyzed. The blank columns in front of the active columns in each row shows the ineffective situations, whereas other blank columns show non-cost effectiveness as performance improvement can be achieved as per standard TAT set by the hospital with the alternative model. The calculation is halted when the length of the queue is ≤ 1.

Table 1.3 Calculation of queuing model Parameters for registration

Timing	**Queuing characteristics**		**No of servers**				
		1	**2**	**3**	**4**	**5**	**6**
7-8am	Av. server utilization{ρ}	0.8	0.4				
	Av. No. of customers in the queue{L_q}	3.2	0.15				
	Av. No. of customers in the server{L}	4	0.95	**INEFFICIENT**			
	Av. waiting period in the queue{W_q}	**37 minutes**	**3 minutes**				
	Av. time period in the system{W}	**60 minutes**	**14 minutes**				
	Probability {% of time} system is empty {P_0}	0.2	0.1				
8-9am							
	Av. server utilization{ρ}		0.97	0.65			
	Av. No. of customers in the queue{L_q}		31.55	0.76			
	Av. No. of customers in the server{L}	**INEFFICIENT**	33.49	2.71			
	Av. waiting duration in the queue{W_q}		**114 minutes**	**5 minutes**	**INEFFICIENT**		
	Av. time period in the system{W}		**120 minutes**	**10 minutes**			
	Probability {% of time} system is empty {P_0}		0.01	0.12			
9-10am							
	Av. server utilization{ρ}		0.95	0.67			
	Av. no. of customers in the queue{L_q}	**INEFFICIENT**	19.58	0.89	**INEFFICIENT**		
	Av. no. of customers in the server{L}		21.49	2.89			
	Av. waiting duration in the queue{W_q}		**51 minutes**	**5 minutes**			
	Av. time period in system{W}		**60 minutes**	**8 minutes**			

	Probability {% of time} system is empty {P_0}		0.02	0.11			
10-11am							
	Av. server utilization{ρ}			0.84	0.63		
	Av. No. of customers in the queue{L_q}	INEFFICIENT		3.82	0.56	INEFFICIENT	
	Av. No. of customers in the server{L}			6.35	3.09		
	Av. waiting duration in the queue{W_q}			**34.8 minutes**	**2.5 minutes**		
	Av. time period in system{W}			**47.8 minutes**	**7 minutes**		
	Probability {% of time} system is empty {P_0}			0.04	0.07		
12-13pm							
	Av. server utilization{ρ}			0.94	0.71		
	Av. No. of customers in the queue{L_q}			15.25	1.07		
	Av. No. of customers in the server{L}	INEFFICIENT		18.08	3.91	INEFFICIENT	
	Av. waiting duration in the queue{W_q}			**29 minutes**	**2.5 minutes**		
	Av. time period in system{W}			**39 minutes**	**9.4 minutes**		
	Probability {% of time} system is empty {P_0}			0.01	0.05		
16-17pm							
	Av. server utilization{ρ}		0.92	0.62			
	Av. No. of customers in the queue{L_q}		10.63	0.60	INEFFICIENT		
	Av. No. of customers in the server{L}	INEFFICIENT	12.48	2.45			
	Av. Waiting duration in the queue{W_q}		**26.5 minutes**	**1.35 minutes**			
	Av. time period in system{W}		**31 minutes**	**6 minutes**			
	Probability {% of time} system is empty {P_0}		0.04	0.14			
19-20pm							

	Av. server utilization{ρ}		0.94	0.63			
	Av. No. of customers in the queue{L_q}		14.60	0.66			
	Av. No. of customers in the server{L}	INEFFICIENT	16.48	2.54	INEFFICIENT		
	Av. Waiting duration in the queue{W_q}		**54 minutes**	**2.96 minutes**			
	Av. time period in system{W}		**60 minutes**	**9.5 minutes**			
	Probability {% of time} system is empty {P_0}		0.03	0.13			
20-21pm							
	Av. server utilization{ρ}		0.89	0.59			
	Av. No. of customers in the queue{L_q}		6.69	0.50			
	Av. No. of customers in the server{L}		8.47	2.28	INEFFICIENT		
	Av. Waiting duration in the queue{W_q}	INEFFICIENT	**27.1 minutes**	**2.5 minutes**			
	Av. time period in system{W}		**38 minutes**	**8 minutes**			
	Probability {% of time} system is empty {P_0}		0.06	0.15			

The alternative model show that during the following peak hours; 7-8am, when the no. of server was 1,the waiting period in queue was found to be 37 minutes while on modifying the number of server to 2, a significant reduction in waiting time could be achieved from 37 minutes to 3 minutes; 8-9am, the number of server was 2 and the waiting time for queue was found to be 114 minutes while on modifying the number of server to 3, a significant reduction in waiting could be achieved from 114 minutes to 5 minutes; 9-10am, the number of server was 2 and waiting time was found to be 51 minutes, on modifying the number of server to 3 the reduction in waiting time could be achieved to 5 minutes; 10-11 am, the number of server was 3 and waiting time was found to be 34.8 minutes, on modifying the no. of server to 4 the reduction in waiting time could be achieved to 2.5 minutes;12-13pm, the no. of server was 3, waiting period was found to be 29 minutes, on modifying the no. of server the waiting period reduction could be achieved to 2.5 minutes; 16-17pm, the number of server was 2, waiting time was found to be 26.5 minutes, on modifying the number to 3 waiting time reduction could be achieved to 1.35 minutes; 19-20pm, the number of server was 2, waiting time was found to be 54 minutes, on modifying the number of server to 3 the waiting reduction could be achieved to 2.96 minutes;20-21pm, the number of server was 2, waiting time was found to be 27.1 minutes and on modifying the number of server to 3, waiting time reduction could be achieved to 2.5 minutes. Considering all the above result in transition to increasing the number of server, there was contrasting finding observed on revising the allocation of server number from 4 to 3 during the afternoon timing that is from 13-14pm. Therefore the queuing model proposed using QM operational research software shows the ideal number of server are required during peak hours for performance improvement at out-patient department

Table 1.4. The number of servers required for registration

Timing	**Total no of patients**	**No of ideal server required**
7-8am	217	**2**
8-9am	466	**3**
9-10am	593	**3**
10-11am	659	**4**
11-12am	727	4
12-13pm	723	4
13-14pm	679	**4/3**
14-15pm	712	3
15-16pm	636	3
16-17pm	648	**3**
17-18pm	595	3
18-19pm	553	3
19-20pm	455	**3**
20-21pm	441	**3**
21-22pm	342	1
22-23pm	160	1
23-00pm	69	1
00-1.00pm	2	1

There may be numerous reasons for this arrival sequence such as conveyance issues, nature of the disease/illness, and convenience of coming early or late and so on. Therefore, the requirement of number of servers is more in morning hours

CHAPTER 5

CONCLUSION AND REFERENCES

Healthcare is a fast growing industry demanding perfection, and optimal utilization of available resources. Not with standing, the trade-off between providing top medical services and keeping the corresponding costs low is evident. This includes the outpatient department in which the interplay between the expense on customers waiting period and the expense on rendering faster services has been depicted by queuing model (34). The study investigated queuing performance of a hospital's outpatient department for four weeks. The objective is to examine the performance level of the existing queuing system and to determine the optimal no. of servers to be required on duty. The findings exhibit capacity utilization rate, number of servers, av. waiting period in queue and server and no. of customers in the queue and server. The peak hours were identified and av. waiting period in queue was calculated. The shrinkage in waiting period based on modification on number of server were identified using queuing softwarewhich is a renowned and tested mathematically system to analyze the waiting lines. It was found that during peak hours, average no. of servers required was four instead of two on existing queuing system. It is worth noting that the analysis uses the multiple servers queuing models {M/M/s} for which are based on the assumptions that arrival rate follows Poisson distribution while the servicing times were exponential in nature and that the infinite calling population is following the first-in-first-out {FIFO} queue discipline. However, actual waiting times were apparently greater than what the results highlighted. This is due to the fact that majority of patients arrived early in the morning while the number of server on duty was usually one. Although results from this study may not be a true reflection of a much longer actual waiting time, management could still leverage the findings as a yardstick for further relevant investigations. After all, the findings may still indicate similar outcomes had it been tested using actual arrival data. For future study, analysis using simulation technique would be valuable to enhance the practicality of the analysis. In all, the study is still beneficial as a decision making aid to the administrator of the outpatient department particularly to re-look and improve existing queuing system for better customer satisfaction According to the result outcome, the no. of servers required for registration can be changed hourly and daily or can be adjusted from other clinics during peak hours. The Av. number of servers required is four compared to previous queuing model where on average, the requirement was only two. According to the model outputs in every section, a briefing is outlined focusing at the marginal changes for every part with respect to the Av. number and waiting time within the queue. Though the result is skewed toward increasing the number of servers, one needs to focus on the expenses involved in accomplishing these marginal changes. More number of executive assigned to the roster cannot be considered as cost effective, thus a balance between the no. of executives, cost and optimal operational performance is utmost important for long term sustainability.

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